# Homework #2

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| **EE 242**  **Spring 2025** | **Yehoshua Luna**  **2322458** |

## Problem #1

, and multiplying by modulates this lowpass signal to be centered at ±4π on the frequency scale. Therefore, should look like the graph below.

A graph of a function

AI-generated content may be incorrect.

Multiplying and gives

Taking the inverse Fourier Transform them yields

1. is simply an impulse train with sample period , so and . Multiplying by gives . Taking the inverse Fourier Transform then yields .

## Problem #2

1. This is a low-pass filter with .
   1. For signals with , this filter simply calculates the derivative of the input, so . This is verified by looking at , which represents a single tone at ±2π. This is inside the range of our low-pass filter, so the signal is not attenuated.
   2. , which is completely outside the range of the low-pass filter, so this signal is completely attenuated, and .

## Problem #3

## A graph of a function AI-generated content may be incorrect.

A graph of a graph of a line

AI-generated content may be incorrect.

## Problem #4

The rectangular pulse train has period T and . It has Fourier coefficients , so we have .

1. Multiplying by a rectangular pulse train with period T and width in the time domain is equivalent to convolving with a sampled sinc function in the frequency domain with sample period . To recover from , we must not have any overlap between each distorted replica of in the frequency domain. Therefore cannot contain any frequencies greater than .
2. The cutoff frequency should be and the gain should be because .

## Problem #5

The ideal low-pass filter then removes the modulator because and amplifies the resulting signal by 2, giving . This shows how phase offset impacts the retrieved signal. When , there is no phase offset, and the resulting signal is an exact replica of . If , then the recovered signal is attenuated by . When , the original signal is totally lost because . Once , the signal becomes inverted, eventually peaking at and then begins oscillating back towards the initial condition when . This cycle repeats indefinitely.

## Problem #6

1. A graph of a function

   AI-generated content may be incorrect.A graph with numbers and lines

   AI-generated content may be incorrect.
2. is a low-pass filter with a single-sided bandwidth of , and has a max frequency of . Multiplication in the time domain is convolution in the frequency domain, so the bandwidths of and are added, giving a a maximum bandwidth of .

## Problem #7

* 1. . This process effectively doubles the bandwidth of to , so we must have a Nyquist sampling rate that follows .
  2. . This doesn’t change the bandwidth of , so .

1. . This sums their bandwidths, creating a signal with new frequency range , so we must have sample rate .
2. Some cursory analysis shows that produces the desired sequence with a sampling rate of . This is because . Adding or subtracting multiples of from shifts this angular frequency by , so other distinct values of that work are and .

## Problem #8

1. The rectangular pulse train has and . It has Fourier coefficients , so we have . limits this to frequencies less than 2 and multiplies it by , so . If , then , so only the case where is left unattenuated by . This means that the output would be . But if , then is completely left alone by , so . (Please be merciful, this was the best that I could do. \*cry\*)

## Problem #9

1. has a maximum of 1000 at . Therefore, over modulation occurs for any values of . So, in order to use envelope detection, we must have , with a minimum of .